String solutions without supersymmetry

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Outline.

Here is an outline of my talk:

Part 1: Introduction

Why look beyond classical string theory?

Classical string theory usually involves specifying a worldsheet conformal field theory.

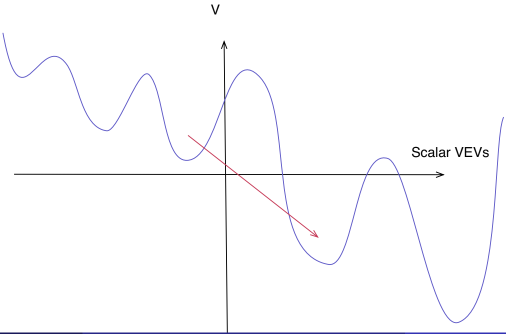
- Such conformal field theories need not describe geometric target spaces or solutions of supergravity.
- However the most heavily studied class of examples are solutions of supergravity like $\mathbb{R}^{3,1} \times CY_3$ or $AdS_5 \times S^5$.
- It is easy to find supersymmetric Minkowski and AdS solutions.

On the other hand, the observed universe exhibits a large dark energy component which constitutes $\sim 70\%$ of the critical density.

The scale of dark energy: $M_{\rm obs} \sim 10^{-30} M_{pl}$.

There are two proposed leading possibilities for explaining this dark energy.

(1) The first is a “landscape” of metastable de Sitter vacua with a rich enough spectrum of cosmological constants to contain our universe (Feng, March-Russell, S.S., Wilczek; Bousso, Polchinski).



(2) The second possibility is dynamical dark energy. A version of this possibility appears in the swampland program under the "refined de Sitter conjecture" which states (Obied, Ooguri, Spodyneiko, Vafa):

$$M_{pl}\frac{|V'|}{V} \equiv \lambda \gtrsim O(1),$$

or

$$-M_P^2\frac{V''}{V} \equiv c^2 \gtrsim O(1).$$

There are no-go results that make an explanation of dark energy in classical string theory very difficult.

We will therefore need to understand something about quantum string theory if we want to explain dark energy, or the definition of quantum gravity in de Sitter spaces, should any such backgrounds exist.

By non-classical string theory or quantum string theory, I mean backgrounds in which the spacetime curvature, or equivalently the spacetime cosmological constant, changes because of string quantum effects.

These are the backgrounds not discussed in standard string theory texts for good reasons.

It is worth dividing quantum string vacua into two types:

- Classical string solutions that receive quantum corrections (worldsheet CFT).
- Solutions of the spacetime equations of motion that have no classical limit as $g_s \to 0$. For example: balancing tree-level stress energy against 1-loop potentials (no classical string theory - most landscape proposals).

The second case is very interesting and has been under more intensive investigation in recent years (Basile, Dudas, Lanza, Mourad, Sagnotti, ...).

We will mainly be concerned with the first case today.

There are also swampland conjectures suggesting there is no AdS/CFT holography for spaces without supersymmetry (Ooguri & Vafa, Freivogel & Kleban). If so, what replaces the CFT?

On the other hand, there are explicit field theory arguments suggesting that suitable CFTs might exist (Giombi & Perlmutter). These constructions involve UV SUSY fixed points so the debate continues.

Today I would like to show that there are examples of quantum string backgrounds with a worldsheet definition and some computational control.

The examples are *AdS* with some amount of quantum uplift. In a quite striking way, the examples *deftly avoid* ever becoming de Sitter.

These *AdS* solutions do not have supersymmetry restored in any obvious limit. Hopefully, they will be useful for exploring holography without supersymmetry.

Part 2: Quantum String Solutions

The basic idea

We would like to address the following questions:

- Are there dS solutions in string theory?
- Are there non-supersymmetric AdS solutions?
- How should we understand non-SUSY AdS backgrounds holographically?

Each of these questions seems to require study of a non-classical string background.

The problem with most non-supersymmetric string backgrounds is that they are unstable.

To solve this problem in this first approach, we will want to construct non-supersymmetric "string islands" with no CFT moduli. That only leaves the dilaton.

Because supersymmetry is broken, there will generically be a 1-loop potential for the dilaton $V_{1-\text{loop}} = \int_{\mathcal{F}} Z_{1-\text{loop}}$.

It is worth noting that for the 3 known non-supersymmetric tachyon-free strings in $D = 10$, this 1-loop potential is positive. As an example: for the $O(16) \times O(16)$ heterotic string it takes the form

$$V_{1-\text{loop}} \sim e^{\frac{5}{2}\phi}.$$

To convert this from a 1-loop potential to a vacuum energy, we will want to stabilize the dilaton. Let's sketch how to do this.

It's useful to first construct an example in $D = 3$ where we have more control. We also have the ability of turning on H_3 through spacetime without breaking Lorentz invariance.

Recall that the well-studied Freund-Rubin solution

$$AdS_3 \times S^3 \times T^4$$

with only NS-flux has an AdS length scale and stabilized string coupling

$$L^2 \sim n_5 \ell_s^2, \qquad e^{2\phi} \sim \frac{n_5}{n_1}.$$

- If we could replace T^4 by a string island that breaks SUSY, we would generate a 1-loop potential independent of the AdS cosmological constant.
- The natural way to do this is to replace T^4 by an asymmetric orbifold T^4/G. Intuition from $D = 10$ would suggest that we might need to consider heterotic models as well as type II to find a case with a positive 1-loop potential.
- Note the dilaton is already massive at tree-level and the string coupling can be made weak!

Current status

Supersymmetric string islands were studied in the past (Dabholkar, Harvey). There is a particularly nice example of the form,

$$\mathbb{R}^{1,5} \times T^4/G, \qquad \Gamma^{(4,4)}(A_4).$$

where $G = \mathbb{Z}_5$ is an asymmetric quotient which freezes all moduli. It breaks all supersymmetry on one side of the string leaving 16 spacetime supersymmetries.

Here $(p_L, p_R) \in \Lambda_W(A_4)$ and $p_L - p_R \in \Lambda_R(A_4)$. The $\mathbb{Z}_5$ action is of the form,

$$|p_L, p_R\rangle \to e^{2\pi i(p_L \cdot v_L - p_R \cdot v_R)}|g_L p_L, g_R p_R\rangle,$$

for a shift vector v. In this case the shift vector is, say, on the right while $g = g_L$ is the Coxeter element of A_4 with order 5 acting on the left.

There are also asymmetric quotients of the $E_8 \times E_8$ heterotic string on T^3 and T^4 which can freeze all Wilson line moduli. These again involve $\mathbb{Z}_5$ and $\mathbb{Z}_6$ actions on the bosonic side together with a symmetric shift in the torus lattice (de Boer et. al.).

These components are characterized by non-trivial CS invariants. For $\mathbb{Z}_5$, $\int_{T^3} \mathrm{CS} = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.

	Λ	$\Lambda^\perp$	Δr	E_t
$\mathbb{Z}_1$	$\Gamma_{3,3} \oplus E_8 \oplus E_8$	$\emptyset$	0	-1
$\mathbb{Z}_2$	$\Gamma_{3,3} \oplus D_4 \oplus D_4$	$D_4 \oplus D_4$	8	$-1/2$
$\mathbb{Z}_3$	$\Gamma_{3,3} \oplus A_2 \oplus A_2$	$E_6 \oplus E_6$	12	$-1/3$
$\mathbb{Z}_4$	$\Gamma_{3,3} \oplus A_1 \oplus A_1$	$E_7 \oplus E_7$	14	$-1/4$
$\mathbb{Z}_5$	$\Gamma_{3,3}$	$E_8 \oplus E_8$	16	$-1/5$
$\mathbb{Z}_6$	$\Gamma_{3,3}$	$E_8 \oplus E_8$	16	$-1/6$

Table 5: Lattices Λ, complements $\Lambda^\perp$, rank reduction Δr and zero-point energies in the twisted sector E_t for the Z_m asymmetric orbifolds corresponding to triples.

I thought it was going to be relatively easy to generalize these models to break all supersymmetry while freezing all moduli and generating no tachyons but all our current attempts in the context of quotients by abelian groups either give moduli or preserve some supersymmetry.

This might be a lamppost effect from studying quotients by abelian groups. Lessons from $c = 1$ CFT suggest generalizing to quotients of T^n/G by non-abelian asymmetric group actions.

That's currently in progress.

There is still an interesting model that looks like it works using the ingredients discussed so far. The model is,

$$AdS_3 \times S^3/\mathbb{Z}_N \times T^4/\mathbb{Z}_5.$$

The action on the torus is the one described by Dabholkar and Harvey.

The action on S^3 is also asymmetric and corresponds to adding Kaluza-Klein monopoles to the background of NS5-branes and F1-strings (Kutasov, Larsen, Leigh).

This $\mathbb{Z}_N$ quotient breaks the other half of the supersymmetries preserved by the $\mathbb{Z}_5$ quotient.

This appears to be a moduli-free non-supersymmetric background with a parameter N controlling the supersymmetry breaking scale. We're in the process of computing the 1-loop potential.

The preceding discussion was in the context of the superstring and the models involve a fairly complicated orbifolding process.

How about non-SUSY strings? Can we find a simpler construction? Let's piggyback off our preceding setup.

There are three known strings in $D = 10$ without tachyons.

- $O(16) \times O(16)$ heterotic string
- type 0'B (Sagnotti)
- $USp(32)$ open string theory (Sugimoto)

Each has a positive dilaton potential $V \sim e^{\alpha\phi}$ with $\alpha > 0$.

Very interesting quantum solutions of the type $AdS \times S$ were studied by several authors (Gubser, Mitra, Dudas, Sagnotti, Murad, Basile, Lanza, ...).

The dilaton equation for NS fields requires something like,

$$\Box\phi - \alpha e^{\alpha\phi} + e^{-\phi}|H|^2 = 0.$$

You can find constant ϕ solutions by balancing the flux against the potential. These can even be weakly coupled for large amounts of flux.

There are typically tachyons below the BF bound in such constructions.

We will try to evade this problem and find a case that might even 'uplift' to de Sitter. Consider

$$AdS_3 \times S^3 \times \hat{S}^3 \times S^1.$$

This only has NS fields turned on. There are 3 integers parametrizing the background:

$$(n_1, n_5, \hat{n}_5).$$

It is well studied in the supersymmetric context. Let's embed it in the $O(16) \times O(16)$ heterotic string.

You might worry right away that we have a circle, a dilaton and Wilson line moduli that might destabilize the background. We will cure these problems.

This background has a CFT worldsheet definition but let's study it from an effective field theory perspective by reducing on $S^3 \times \hat{S}^3 \times S^1$.

The tree-level action for the $O(16) \times O(16)$ heterotic string takes the form:

$$S_{D=10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, e^{-2\phi} \sqrt{-g} \left(R + 4(\partial\phi)^2 - \frac{1}{12}|H_3|^2 + \dots \right),$$

We take the following $D = 10$ metric,

$$ds^2 = ds^2_{\mathcal{M}_3} + e^{2\chi} d\Omega_3^2 + e^{2\hat{\chi}} d\hat{\Omega}_3^2 + e^{2\sigma} dx_{10}^2,$$

where the spheres have volume $2\pi^2 L^3$, $2\pi^2 \hat{L}^3$ and the circle has radius r.

We turn on flux through both spheres:

$$\frac{1}{4\pi^2\alpha'}\int_{S^3} H_3 = n_5, \qquad \frac{1}{4\pi^2\alpha'}\int_{\hat{S}^3} H_3 = \hat{n}_5, \qquad n_5, \hat{n}_5 \in \mathbb{Z}.$$

We also need electric $H_3^{\rm electric}$,

$$H_3^{\rm electric} = 8\pi\alpha'^3 g_s^2 \frac{n_1}{rL^3\hat{L}^3} e^{2\phi-\sigma-3\chi-3\hat{\chi}}\,\epsilon_3.$$

Minimizing the tree-level potential potential gives,

$$L = \sqrt{\alpha' |n_5|}, \qquad \hat{L} = \sqrt{\alpha' |\hat{n}_5|}, \qquad \frac{g_s^4}{r^2} = \frac{n_5^2 \hat{n}_5^2 (|n_5| + |\hat{n}_5|)}{16\pi^2 \alpha' n_1^2},$$

with cosmological constant:

$$\Lambda = -\left(\frac{1}{L^2} + \frac{1}{\hat{L}^2}\right) = -\frac{1}{\alpha'}\left(\frac{1}{|n_5|} + \frac{1}{|\hat{n}_5|}\right).$$

Note that the string coupling can be made arbitrarily small independently of the value of Λ,

$$g_s \to 0, \qquad n_1 \to \infty.$$

This is identical to the SUSY analysis so there should be no BF-violating tachyons in the story at all.

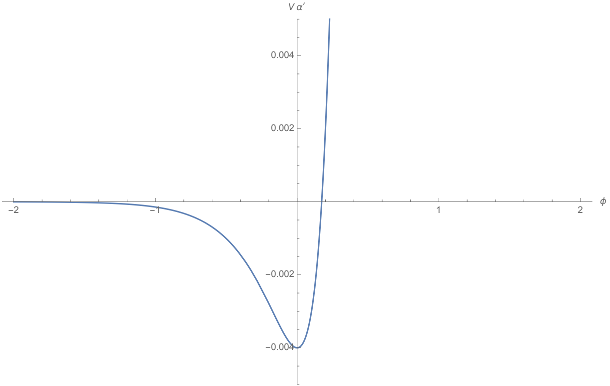


Figure: The potential $V(\phi)$ with $n_5 = \hat{n}_5 = 10^3$.

The string 1-loop potential on $\mathbb{R}^9 \times S^1$ was studied long ago numerically (Ginsparg & Vafa). It has two key features. The radius of the circle is frozen at the self-dual value,

$$r = \sqrt{\alpha'}.$$

The σ field is now massed up. At this point, the gauge symmetry is enhanced to $SO(16) \times SO(16) \times SU(2)$. Fortunately all the Wilson line moduli are also massive at this point so we can ignore them.

There are no massless moduli left!

Compactifying to $D = 3$ on $S^3 \times \hat{S}^3$ gives:

$$V^{\mathrm{1-loop}} = e^{6\phi - 3\sigma - 9\chi - 9\hat{\chi}}\Lambda_3, \qquad \Lambda_3 = \lambda \times \frac{g_s^2}{\alpha'} = 0.576 \times \frac{g_s^2}{\alpha'}\,.$$

Now something amazing happens. Suppose we treat this as a small pertubation and evaluate at the tree-level minimum of the potential then we can always uplift to de Sitter by choosing $n_5 = \hat{n}_5$ for simplicity and

$$n_5 > \frac{3.472}{g_s^2}.$$

If, on the other hand, we compute the slightly perturbed new minimum of the combined tree and 1-loop potential, we find instead:

$$
\begin{aligned}
g_{s,\circ}^2 =& \frac{|n_5 \hat{n}_5| \sqrt{(|n_5| + |\hat{n}_5|)}}{4\pi |n_1|} \left(\sqrt{1+\beta^2} - \beta \right) = g_s^2 \left(\sqrt{1+\beta^2} - \beta \right) \\
\beta :=& \frac{3\lambda |n_5 \hat{n}_5|^3}{64\pi |n_1| \sqrt{|n_5| + |\hat{n}_5|}} \,.
\end{aligned}
$$

Note that in the $\beta \to 0$ limit ($n_1^2 \gg n_5^5 \hat{n}_5^6, n_5^6 \hat{n}_5^5$), we recover the tree level g_s.

The cosmological constant can never become positive!

$$V_\circ = \left(-\frac{4}{\alpha' |n_5|} - \frac{4}{\alpha' |\hat{n}_5|} \right) + \frac{32\pi^2 n_1^2}{\alpha' |n_5 \hat{n}_5|^3} g_s^4 \left(\sqrt{1+\beta^2} - \beta \right) + \frac{\lambda}{\alpha} g_s^2 \left(\sqrt{1+\beta^2} - \beta \right) .$$

This appears to be true even if we pretend the one-loop coefficient λ is enormous. It doesn't want to uplift to de Sitter.

To summarize. We have a completely non-supersymmetric *AdS* solution.

- It is has a worldsheet description that can be studied.
- There is a tachyon at the BF bound in the SUSY theory (Eberhardt, Gaberdiel, Gopakumar, Li).

  Our current analysis suggests this tachyon moves above the BF bound in the non-SUSY theory. This indicates perturbative stability.
- The worldsheet correlators should tell us about holography.

There might be other instabilities involving the condensation of multi-trace operators, but those are harder to see from a bulk description (Klebanov et. al.). There will almost definitely be non-perturbative instabilities.

There appears to be much more to learn in the arena of tachyon-free non-SUSY strings.

However, de Sitter space (if it exists) appears to require something more than a Casimir energy at least in string theory.